

$$\textcircled{1} \quad \int \sqrt{x} \ln x \, dx = \int x^{\frac{1}{2}} \ln x \, dx = \left. \begin{array}{l} u = \ln x \\ u' = x^{-\frac{1}{2}} \end{array} \right\} \begin{array}{l} u' = \frac{1}{x} \\ v = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} x^{\frac{3}{2}} \end{array}$$

$$= \ln x \cdot \frac{2}{3} x^{\frac{3}{2}} - \int \frac{1}{x} \cdot \frac{2}{3} x^{\frac{3}{2}} \, dx =$$

$$= \ln x \cdot \frac{2}{3} x \sqrt{x} - \frac{2}{3} \int x^{\frac{1}{2}} \, dx =$$

$$= \frac{2}{3} x \sqrt{x} \ln x - \frac{2}{3} \frac{x^{\frac{3}{2}}}{\frac{3}{2}} =$$

$$= \frac{2}{3} x \sqrt{x} \ln x - \frac{4}{9} x \sqrt{x} + c =$$

$$= \frac{2}{3} x \sqrt{x} \cdot \left(\ln x - \frac{2}{3} \right) + c$$
