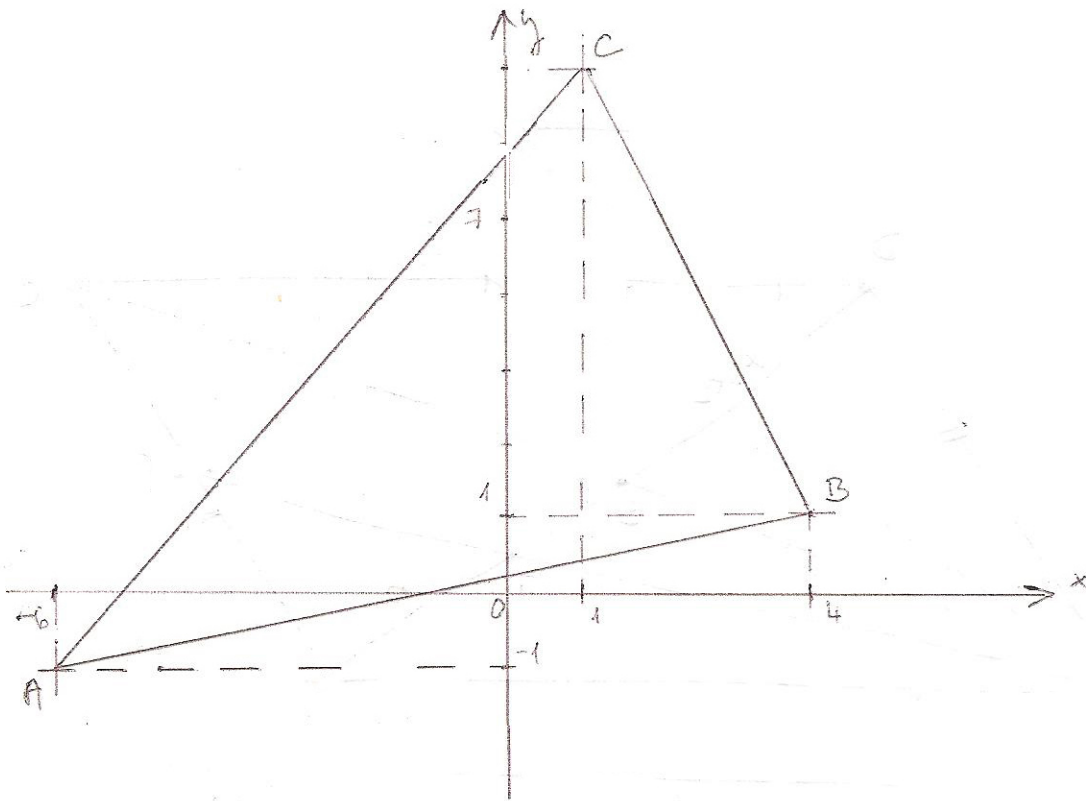


$$\Delta ABC: A[-6; -1] \quad B[4; 1] \quad C[1; 7]$$



$$1) \quad \sigma_{\Delta ABC} = ?$$

$$\sigma = a + b + c$$

$$\sigma = |BC| + |AC| + |AB|$$

$$|BC| = \sqrt{(b_1 - c_1)^2 + (b_2 - c_2)^2}$$

$$|BC| = \sqrt{3^2 + 6^2}$$

$$|BC| = \sqrt{45} = \underline{3\sqrt{5}} \doteq 6,71$$

$$45 = 5 \cdot 9$$

$$|AC| = \sqrt{(a_1 - c_1)^2 + (a_2 - c_2)^2}$$

$$|AC| = \sqrt{7^2 + 8^2}$$

$$|AC| = \sqrt{113} \doteq 10,63$$

$$|AB| = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

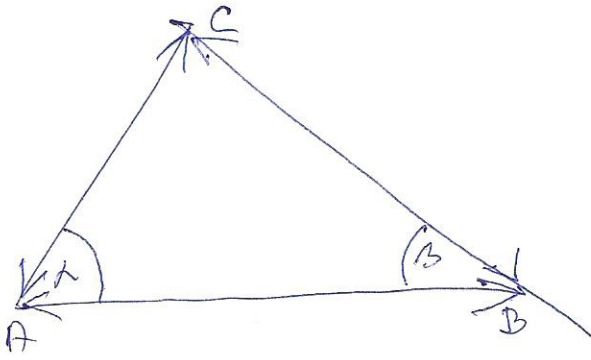
$$|AB| = \sqrt{10^2 + 2^2}$$

$$|AB| = \sqrt{104} = \underline{2\sqrt{26}} \doteq 10,20$$

$$104 = 4 \cdot 26$$

$$\sigma_{\Delta ABC} = \underline{3\sqrt{5}} + \sqrt{113} + \underline{2\sqrt{26}} \doteq 6,71 + 10,63 + 10,20 = \underline{27,54}$$

2) $\alpha, \beta, \gamma = ?$



$$\vec{CA} = A - C = (-7; -8)$$

$$\vec{CB} = B - C = (3; -6)$$

$$\vec{AB} = B - A = (10; 2)$$

$$\vec{AC} = C - A = (7; 8)$$

$$\vec{BA} = A - B = (-10; -2)$$

$$\vec{BC} = C - B = (-3; 6)$$

$$\alpha = \varphi(\vec{AB}, \vec{AC})$$

↑
odchytk vektoru

$$\begin{aligned} \cos \alpha &= \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{10 \cdot 7 + 2 \cdot 8}{\sqrt{10^2 + 2^2} \cdot \sqrt{7^2 + 8^2}} \\ &= \frac{70 + 16}{\sqrt{104} \cdot \sqrt{113}} = \frac{86}{\sqrt{104 \cdot 113}} \\ &\doteq 0,793 \Rightarrow \alpha \doteq \underline{\underline{37^\circ 30'}} \end{aligned}$$

$$\beta = \varphi(\vec{BA}, \vec{BC})$$

$$\begin{aligned} \cos \beta &= \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| \cdot |\vec{BC}|} = \frac{-10 \cdot (-3) + (-2) \cdot 6}{\sqrt{10^2 + 2^2} \cdot \sqrt{3^2 + 6^2}} \\ &= \frac{30 - 12}{\sqrt{104} \cdot \sqrt{45}} = \frac{18}{\sqrt{104 \cdot 45}} \\ &\doteq 0,263 \Rightarrow \beta \doteq \underline{\underline{74^\circ 45'}} \end{aligned}$$

$$\gamma = \varphi(\vec{CA}, \vec{CB})$$

$$\begin{aligned} \cos \gamma &= \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}| \cdot |\vec{CB}|} = \frac{-7 \cdot 3 + (-8) \cdot (-6)}{\sqrt{7^2 + 8^2} \cdot \sqrt{3^2 + 6^2}} \\ &= \frac{-21 + 48}{\sqrt{113} \cdot \sqrt{45}} = \frac{27}{\sqrt{113 \cdot 45}} \\ &\doteq 0,379 \Rightarrow \gamma \doteq \underline{\underline{67^\circ 45'}} \end{aligned}$$

Zk: $\boxed{\alpha + \beta + \gamma = 180^\circ}$

$$37^\circ 30' + 74^\circ 45' + 67^\circ 45' = 178^\circ 120' = \underline{\underline{180^\circ}}$$

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$$S_{\triangle ABC} = ?$$

$$S = \frac{1}{2} ab \sin \gamma$$

$$a = |BC| = 2\sqrt{26} \doteq 10,20 \quad (\text{viz 1. p\u0159.})$$

$$b = |AC| = \sqrt{113} \doteq 10,63 \quad -11-$$

$$\gamma \doteq 67^\circ 45'$$

$$S = \frac{1}{2} \cdot 2\sqrt{26} \cdot \sqrt{113} \cdot \sin 67^\circ 45' \doteq \underline{\underline{50,17}}$$

4) strany a, b, c a) parametricky

strana a:

$$a = \overleftrightarrow{BC} : X = B + \lambda \cdot \overrightarrow{BC}$$

$$\begin{cases} x = 4 - 3\lambda + 2 \\ y = 1 + 6\lambda \end{cases} \quad \lambda \in \mathbb{R}$$

b) obecn\u011b: $2x + y = 9$

a: $2x + y - 9 = 0$

strana b:

$$b = \overleftrightarrow{AC} : X = A + \lambda \cdot \overrightarrow{AC}$$

$$\begin{cases} x = -6 + 7\lambda \cdot 8 \\ y = -1 + 8\lambda \cdot (-7) \end{cases} \quad \lambda \in \mathbb{R}$$

$$8x - 7y = -48 + 7$$

b: $8x - 7y + 41 = 0$

... parametricky

... obecn\u011b

střana c:

$$c = \overleftrightarrow{AB} \quad : \quad X = A + \lambda \cdot \overrightarrow{AB}$$

$$\parallel \quad x = -6 + 10\lambda$$

$$\parallel \quad y = -1 + 2\lambda \quad (\cdot (-5)) \quad \lambda \in \mathbb{R} \quad \dots \text{param.}$$

$$x - 5y = -6 + 5$$

$$c: \quad \underline{\underline{x - 5y + 1 = 0}} \quad \dots \text{obecně}$$

5) směrnice tržer a

$$a: \quad 2x + y - 9 = 0 \quad (\text{viz 4})$$

\Downarrow

$$a: \quad \underline{\underline{y = -2x + 9}}$$

6) $T_{\triangle ABC} = ?$

$T \in \Delta_a \cap \Delta_b$

$\Delta_a = AS_a$

$A[-6|-1]$

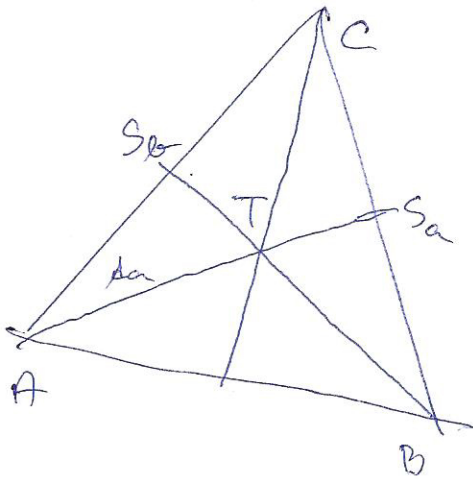
$S_a = \frac{B+C}{2} = \left[\frac{5}{2} | 4\right]$

$\vec{p}_{\Delta_a} = \vec{AS_a} = S_a - A = \left(\frac{17}{2} | 5\right)$

$\Delta_a: X = A + \lambda \cdot \vec{AS_a}$

$x = -6 + \frac{17}{2}\lambda \quad | \cdot 10$

$y = -1 + 5\lambda \quad | \cdot 17$



$10x - 17y = -60 + 17$

$\Delta_a: 10x - 17y + 43 = 0$

$\Delta_b = BS_b$

$B[4|1]$

$S_b = \frac{A+C}{2} = \left[-\frac{5}{2} | 3\right]$

$\vec{p}_{\Delta_b} = \vec{BS_b} = S_b - B = \left(-\frac{13}{2} | 2\right)$

$\Delta_b: X = B + \lambda \cdot \vec{BS_b}$

$x = 4 - \frac{13}{2}\lambda \quad | \cdot 4$

$y = 1 + 2\lambda \quad | \cdot 13$

$4x + 13y = 16 + 13$

$\Delta_b: 4x + 13y - 29 = 0$

(6 pts.)

$T \in \Delta_a \cap \Delta_b$:

$$\Delta_a: 10x - 17y + 43 = 0 \quad | \cdot (-2)$$

$$\Delta_b: 4x + 13y - 29 = 0 \quad | \cdot 5$$

$$34y + 65y - 86 - 145 = 0$$

$$99y = 231$$

$$y = \frac{231}{99}$$

$$\underline{\underline{y = \frac{7}{3}}}$$

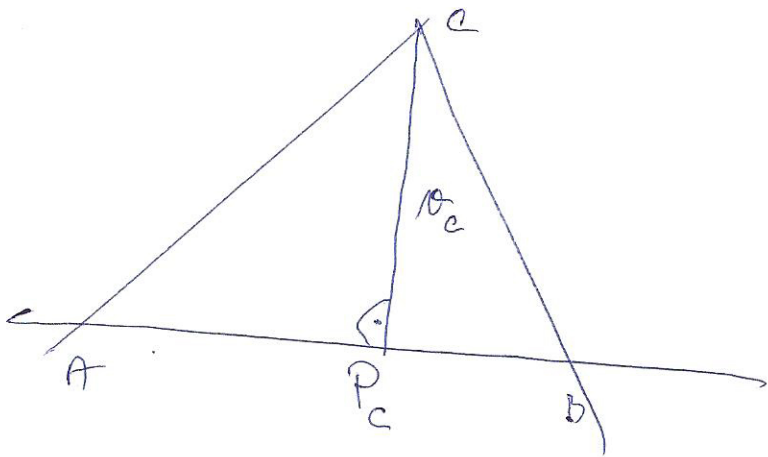
$$4x + 13 \cdot \frac{7}{3} - 29 = 0$$

$$4x = -\frac{4}{3}$$

$$\underline{\underline{x = -\frac{1}{3}}}$$

$$\underline{\underline{T \left[-\frac{1}{3} \mid \frac{7}{3} \right]}}$$

7) $|v_a|, |v_b|, |v_c| = ?$



$C[1;7]$

$\overleftrightarrow{AB}: x - 5y + 1 = 0$ (viz 4)

$$|v_c| = |C; \overleftrightarrow{AB}| = \frac{|1 - 5 \cdot 7 + 1|}{\sqrt{1^2 + 5^2}} = \frac{|-33|}{\sqrt{26}} = \frac{33}{\sqrt{26}} \doteq \underline{\underline{6,47}}$$

$B[4;1]$ $\overleftrightarrow{AC}: 8x - 7y + 41 = 0$ (viz 4)

$$|v_b| = |B; \overleftrightarrow{AC}| = \frac{|8 \cdot 4 - 7 \cdot 1 + 41|}{\sqrt{8^2 + 7^2}} = \frac{66}{\sqrt{113}} \doteq \underline{\underline{6,21}}$$

$A[-6;-1]$ $\overleftrightarrow{BC}: 2x + y - 9 = 0$ (viz 4)

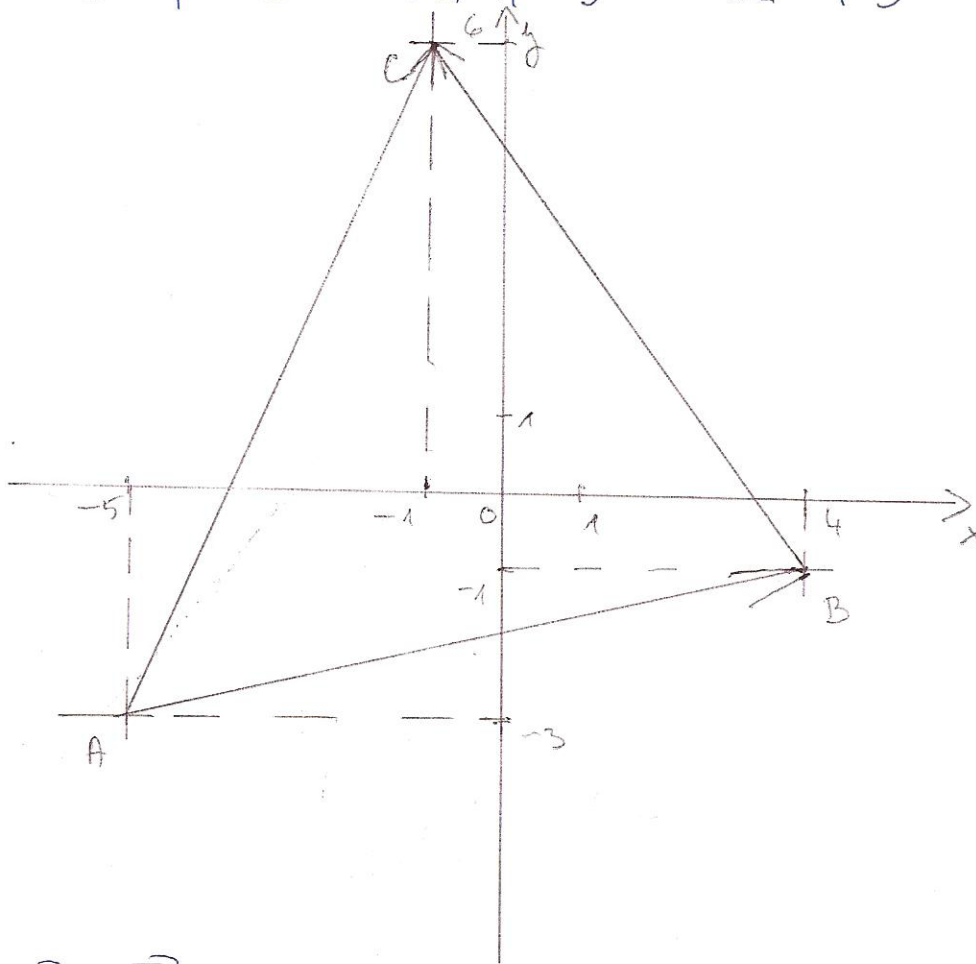
$$|v_a| = |A; \overleftrightarrow{BC}| = \frac{|2 \cdot (-6) - 1 - 9|}{\sqrt{2^2 + 1^2}} = \frac{|-22|}{\sqrt{5}} = \frac{22}{\sqrt{5}} \doteq \underline{\underline{9,84}}$$

1)

$$A[-5; -3]$$

$$B[4; -1]$$

$$C[-1; 6]$$



$$a) \quad \vec{u} = \vec{AB} = B - A = (9; 2)$$

$$\vec{v} = \vec{BC} = C - B = \underline{\underline{(-5; 7)}}$$

$$\vec{w} = \vec{AC} = C - A = \underline{\underline{(4; 9)}}$$

$$b) \quad \vec{a} \cdot \vec{b} = ?$$

$$\vec{a} = 2\vec{u} - \vec{v}$$

$$\vec{b} = 2\vec{v} + 3\vec{w}$$

$$\vec{a} = 2 \cdot (9; 2) - (-5; 7) = \underline{\underline{(23; -3)}}$$

$$\vec{b} = 2 \cdot (-5; 7) + 3(4; 9) = \underline{\underline{(2; 41)}}$$

$$\vec{u} = (9; 2) \quad (\text{v. 2a})$$

$$\vec{v} = (-5; 7)$$

$$\vec{w} = (4; 9)$$

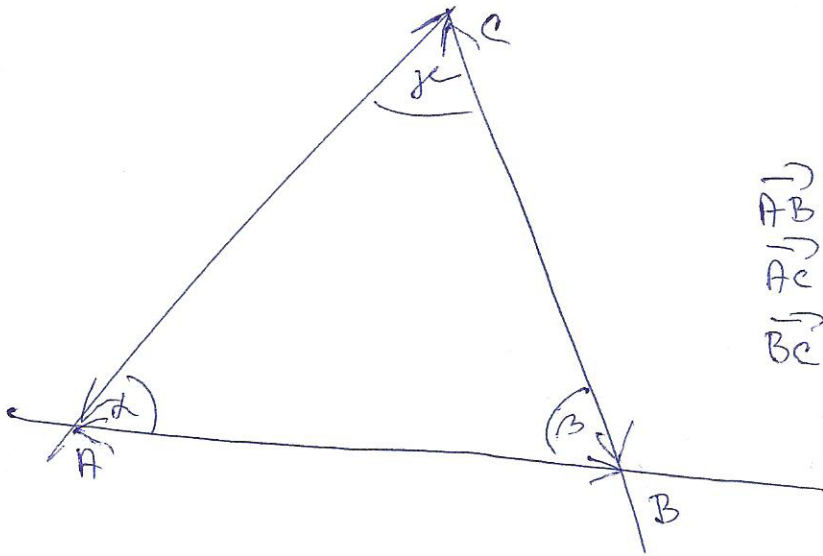
$$\vec{a} \cdot \vec{b} = (23; -3) \cdot (2; 41) = 23 \cdot 2 - 3 \cdot 41 = \underline{\underline{-77}}$$

2) $\alpha, \beta, \gamma = ?$

$$A[-5; -3]$$

$$B[4; -1]$$

$$C[-1; 6]$$



$$\begin{aligned} \vec{AB} &= B - A = (9; 2) & \vec{BA} &= (-9; -2) \\ \vec{AC} &= C - A = (4; 9) & \vec{CA} &= (-4; -9) \\ \vec{BC} &= C - B = (-5; 7) & \vec{CB} &= (5; -7) \end{aligned}$$

$$\alpha = \varphi(\vec{AB}; \vec{AC})$$

$$\begin{aligned} \cos \alpha &= \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{9 \cdot 4 + 2 \cdot 9}{\sqrt{9^2 + 2^2} \cdot \sqrt{4^2 + 9^2}} = \\ &= \frac{54}{\sqrt{85} \cdot \sqrt{97}} \doteq 0,595 \Rightarrow \alpha \doteq \underline{\underline{53^\circ 31'}} \end{aligned}$$

$$\beta = \varphi(\vec{BA}; \vec{BC})$$

$$\begin{aligned} \cos \beta &= \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| \cdot |\vec{BC}|} = \frac{-9 \cdot (-5) + (-2) \cdot 7}{\sqrt{9^2 + 2^2} \cdot \sqrt{5^2 + 7^2}} = \\ &= \frac{31}{\sqrt{85} \cdot \sqrt{74}} \doteq 0,391 \Rightarrow \beta \doteq \underline{\underline{66^\circ 59'}} \end{aligned}$$

$$\gamma = \varphi(\vec{CA}; \vec{CB})$$

$$\begin{aligned} \cos \gamma &= \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}| \cdot |\vec{CB}|} = \frac{-4 \cdot 5 + (-9) \cdot (-7)}{\sqrt{4^2 + 9^2} \cdot \sqrt{5^2 + 7^2}} = \\ &= \frac{43}{\sqrt{97} \cdot \sqrt{74}} \doteq 0,508 \Rightarrow \gamma \doteq \underline{\underline{59^\circ 30'}} \end{aligned}$$

$$\text{Zk: } \boxed{\alpha + \beta + \gamma = 180^\circ}$$

$$53^\circ 31' + 66^\circ 59' + 59^\circ 30' = 179^\circ 120' = \underline{\underline{180^\circ}}$$